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ABSTRACT

In this study, we examined the introduction of ratio and proportion concepts in six textbook series from four different regions, China, Japan, Taiwan and U.S. When analyzing the definition of ratio and equal ration as well as the types of ratio and proportion application problems included in each textbook series, we found similarities and differences among these textbook series both within and across regions. Future studies will be needed to examine these differences further. (Author)

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A COMPARATIVE STUDY OF THE SELECTED TEXTBOOKS FROM CHINA, JAPAN, TAIWAN AND THE UNITED STATES ON THE TEACHING OF RATIO AND PROPORTION CONCEPTS

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Abstract: In this study, we examined the introduction of ratio and proportion concepts in six textbook series from four different regions, China, Japan, Taiwan and U.S. When analyzing the definition of ratio and equal ratio as well as the types of ratio and proportion application problems included in each textbook series, we found similarities and differences among these textbook series both within and across regions. Future studies will be needed to examine these differences further.

The results of cross-national comparisons of mathematical performance such as TIMSS and TIMSS-R have received considerable attention by the educational research community as well as the general public. This type of analyses provide unique opportunities to understand the current state of students' learning and suggest ways future learning can be supported and enhanced (Cai, 2001). One of the findings from previous cross-national studies was that, in general, the U.S. students did not perform as well as the Asian students in mathematics. Because of the complexity of interpreting cross-national differences, we are just beginning to understand the possible factors that may contribute to the differences in mathematics. In an earlier study, we examined curricular treatments of arithmetic average in U.S. and Asian school mathematics. (Cai, Lo, & Watanabe, 2001). In this study, we focused on how the textbooks and teacher manuals were designed to facilitate students' understanding of the ratio and proportion concepts.

Theoretical Framework

Cross-national comparison of textbooks

Cross national comparisons of mathematics textbook are important because U.S. textbooks constitute a de facto national curriculum (Mayers, Sims, & Tajika, 1995). Therefore, examining the content and teaching methods of the textbooks provides a partial account of how mathematics is taught in different regions. For example, Mayers, Sims & Tajika (1995) compared the lesson on addition and subtraction of signed whole numbers in three seventh-grade Japanese and four U.S. mathematics textbooks. They found that Japanese books contained many more worked-out examples and relevant illustrations than did the U.S. books. And the U.S. books contained

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roughly as many exercises and many more irrelevant illustrations than did the Japanese textbooks. When comparing selected first-grade Japanese and U.S. textbooks, Samimy and Liu (1997) found that while the Japanese textbook was mainly characterized by its short explanation and equal distribution of both computational and word problems, the U.S. textbook was characterized by its detailed explanations, repetitions and pictures. Li (2000) compared the problems presented in selected U.S. and China middle school mathematics textbooks on the topic of addition and subtraction integers through three dimensions: mathematical feature, contextual feature and performance requirements. He found some striking difference between the U.S. and the China textbooks with respective to problems' performance requirement. For example, more problems in the U.S. textbooks (19%) required mathematical explanations than the problems in China textbooks (0%). All these analyses have identified important differences between the selected U.S. textbooks and Asian textbooks that might provide some explanations of the performance differences in the international comparative studies such as TIMSS and TIMSS-R. However, none of the above studies have examined textbooks' attempts to support students' learning of a major mathematical concept. Our previous study showed that such investigation was informative for identifying the difference in curricular emphases that might explain the difference in student performance (Cai, Lo, & Watanabe, 2001). The present study intended to provide further information by examining the textbooks' attempts to support students' learning of the ratio and proportion concepts by several U.S. and Asian textbook series.

Ratio and proportion concepts

We selected the ratio and proportion concepts to be the focus of this study because of their importance in school mathematics. Typically, students received their initial instruction on ratio and proportion as early as the fourth or fifth grade. Then they continued to study these topics with increasing complexity and formality throughout the middle school years. When solving ratio and proportion problems, students applied their concepts on multiplication and division of whole number and rational numbers. This process helped students build a solid foundation for more advanced mathematics concepts such as algebraic reasoning and analytic geometry. There has been a consensus that the concepts of ratio and proportion developed through stages. It took several years to develop the formal reasoning and computation skills that could be flexibly applied to various number structures and across different problem contexts (Lo & Watanabe, 1997; Vergnaud, 1988).

One challenge of teaching and learning of ratio and proportion concepts was the wide variety of problem types and contexts as well as number structures that could influence the students' performance (Harel, Behr, Post, & Lesh, 1991; Kaput & West, 1994; Lamon, 1993). In order to compare different textbooks within mathematics education research literature on ratio and proportion, this study examined the amount of variety included in the examples and exercises in each textbook series.

Method

The selection of the typical Asian mathematics textbook series in China, Japan and Taiwan was not difficult because all textbook series in these regions must conform to the curriculum set by the Ministry of Education at each region. The Asian textbook series chosen for this study included China series (Division of Mathematics, 1996), Japan series (Tokyo Shoseki, 1998) and Taiwan series (National Printing Office, 1999).

Finding a representative U.S. textbook, on the other hand, was much more difficult due to the lack of national curricular guidelines. In order to reflect this diversity, we decided to include both National Science Foundation funded "reform" curricular and "commercial" curriculum in our study. The two reforms curricular were *Mathematics in Context* (National Center for Research in Mathematical Sciences Education at the University of Wisconsin/Madison and Freudenthal Institute a the University Utrecht, 1997-1998), and *Connected Mathematics* (Fay, et. al, 1998). Because there were a wide variety of "commercial" textbooks available in the market, it was quite impossible to find a "typical curriculum" in the same sense as the Asian textbooks. As a result, we used a series that was adopted by a major school district in a Mid-Atlantic state, *Math in My World* (Clements, et. al., 1999).

Because of the developmental nature of the ratio and proportion concepts, it was necessary to examine the teaching of these concepts across several grade levels. Three levels of analysis were planned. First, we examined how the ratio concept was introduced by each textbook series through the examinations of the goals, examples, exercises presented by both the student books and the teacher manuals. Second, we studied the depth of the textbook presentation on ratio and proportion concepts by the type of connections to the other units in the same textbook series as identified by the teachers' manual. However, we found that frequently, for the ease of use, the teacher manuals presented only the vertical connections of a particular unit within a few grade spans, but not the horizontal connections. Therefore, we decided to also identify "other" ratio-related units there were introduced in the curriculum series but not identified by the teacher's edition as connected units. The goal was to capture the richness of the textbook presentation on ratio and proportion concepts. After these analyses were conducted for each textbook series, comparisons were made to highlight both the similarities and differences among these six textbook series. In this paper, we will present some results about the first level of the analysis, focusing on the introduction of ratio and proportion concepts.

Analysis

First, we will summarize the lessons that have been included in this analysis from each textbook series. The ratio concepts were formally introduced in all three Asian textbook series at the sixth grade. In China series, the concept of ratio (3 lessons) was introduced at the first half of the sixth grade within the unit of "Division with frac-

tions". The unit on proportion that include direct proportion, inverse proportion, map reading and other application of proportion was introduced at the end of sixth grade (10 lessons). Similar to China's series, the ratio was introduced at the first half of the sixth grade (8 lessons) in Taiwan series. Proportion-related concepts were introduced in the second half of the sixth grade in three different units: direct proportion (6 lessons), inverse proportions (7 lessons) and scale drawing/map reading (8 lessons). In Japan series, the concept of ratio was introduced in the sixth grade (8 lessons), followed by lessons on scale drawing and map readings (9 lessons). The concept of proportion including direct and inverse proportion was introduced about three months later at the same grade (12 lessons). There were some rate-related problems embedded in a unit called "per-unit-quantity" in the fifth grade, prior to the formal introduction of ratio and proportion concepts.

In the U.S. commercial *Math in My World*, the ratio concept was first mentioned briefly together with the introduction of rate in the fifth grade (4 lessons). This topic was revisited with more elaboration and additional topics such as rate, proportion and scale drawings in the sixth grade (12 lessons). Different from the three Asian textbooks and the U.S. commercial, the definition of ratio in the *Connected Mathematics* was given explicitly only after the students were given some chance to explore the idea of "ratio" informally. The term "ratio" was first mentioned in the unit of "Stretching and Shrinking" (seventh grade, 22-23 lessons). Students were asked to investigate the relationship between length segments and angles of different computer-generated figures in order to study the idea of similar figures. In a subsequent unit "Comparing and Scaling" (seventh grade, 2-24 lessons) students revisit the idea of ratio together with the concepts of rate and percents in various non-geometrical contexts. Similar to the *Connected Mathematics*, the definition of ratio was given explicitly only after the students were given some chance to explore the idea of "ratio" informally in the *Mathematics in Context*. The concept of ratio was first introduced informally through the unit "Some of the Parts" a unit on Fraction (Grade 5/6, 3 lessons). Ratio table was introduced as a model use to increase or decrease the serving sizes of recipes, and later became a tool for organizing fraction calculation. The idea of ratio was continuously explored informally in the context of percent, decimals, and fraction in the unit of "Per Sense" in grade 5/6 (14 lessons). And in the unit of "Ratios and Rates" in grade 6/7 (19 lessons), the ideas of ratio and rates were explored more formally and fully. Concepts and terminology like "part-part ratio" and "part-whole ratio," "constant and variable relationship" were introduced to help students further explore the relationship among ratios, fractions, percents and decimals, as well as among ratios, rate and average. In the following, we will present the initial analysis in four sections "definition of ratio," "equal ratio," "application of ratio" and "application of proportion."

Definition of ratio

In Chinese, the word used for ratio is "bi." It means "to compare or comparison." When used in mathematical context, the multiplicative comparison of two quantities was emphasized. For example, in China series, the ratio was introduced by comparing the length and width of a flag, "Figure out the length of the flag is how many times as the width of the flag?" In Taiwan series, the ratio was introducing by comparing the number of cookies each of the two brothers have, "Figure out the number of big brother's cookies is how many times as the little brother's cookies." In both series, the equality notation a:b=a÷b=a/b was introduced. The notation "a:b" is called "ratio of a and b" or "a to b" and the "a/b" is called the "value of ratio." The relationship among ratio, division and fraction were emphasized through this equality. In Taiwan series, the ratio was restricted to the relationship between two quantities with the like measures, but in China series, examples of ratio included "distance: hours" of different measures.

In the Japanese textbook, the ratio concepts was introduced by asking students to determine whether the taste of the two dressings, one made at home and the other made at the school, would taste the same. The dressing made at home consisted of 2 spoons of vinegar and 3 spoons of oil. The one made at school consisted of 8 spoons of vinegar and 12 spoons of oil. Compared to both China and Taiwan series, the Japan series made more clearly distinction between the "ratio" and the "value of ratio." For example, the following paragraph was included in the teacher manual: "A ratio is a representation of the proportional relationship between two quantities using two whole numbers. On the other hand, the value of a ratio, that is, A is 2/3 of B, expresses the amount of A using one number when we consider B as 1. Thus, when we express the proportional relationship of two quantities by an ordered pair of two whole numbers, we have a ratio, while we express that proportional relationship using one number, we have the value of a ratio."

All three U.S. textbook series defined ratio as a way to relate two numbers: for so many of the first quantity, there were so many of the second quantity. Students were taught three ways of writing ratios: 2 to 3, 2:3 or 2/3. The idea of "multiplicative relationship" was not emphasized explicitly. Different from all three Asian textbook, there was no clear distinction made between "ratio" and "the value of ratio." They simply made 2:3 equivalent to 2/3, without bridging through division operation.

Equal ratio

China textbook series introduced the idea of equal ratio directly through the previous work on fractions and equivalent fractions. The unit of Equal Ratio started out with the following question, "We have learned the invariant property of quotient in division and the basic property of fractions, if we combine these two properties, think about it, what properties about ratio do we have?" The following principle was listed

after the question: When multiplying or dividing both the front term and the back term by the same number (other than zero), the value of the ratio does not change.

Both Taiwan and Japan series gave detailed illustrations to connect the idea of equal ratio with the principle of outlined above by China series. For example, in Taiwan series, a ratio of "20:30" was first identified as the relationship between the width (20 cm) and the length of a rectangle (30 cm). Then the students were asked to use 5 cm as a unit to measure the width and the length of the same rectangle. As a result, the width became 4 units (of 5 cm) and the length became 6 units (of 5 cm), thus a ratio "4:6" can be used to represent the same width vs. length relationship. Lastly, the students were asked to use 10 cm as a unit to measure the width and the length of the same rectangle, and another ratio "2:3" was obtained. From here the relationship 20:30 = 4:6 = 2:3 was established. Two more similar examples were shown before the algorithmic principle of equal ratio was introduced in Taiwan series. Similar discussion and illustration were used in Japanese series.

Even though China series did not include pictures to facilitate the explanation, it contained the most variety of problems in their exercise sections that challenged students' thinking. For example, one exercise asked "If Li Ming is 1 meter tall, and his dad is 173 cm tall. Li Ming said that the ratio between his height and his dad's height is 1: 173. Is this correct? What do you think the ratio should be? " This exercise pointed to the students the importance of converting both quantities to the same measuring unit when forming a ratio relationship

The *Math in My World* introduced the idea of equal ratio by asking students to use two red counters and three yellow counters to represent the ratio "2:3." Then repeatedly added groups of two red counters and three yellow counters to generate equal ratios such as "4:6" "6:9" "8:12" etc. The idea of ratio table was introduced as one way to find equal ratios. Another way of finding equal ratio was to multiply or divide the numerator and denominator by the same number. No connections were made between these two ways of finding equal ratios.

The *Connected Mathematics* introduced the idea of equal ratio through a series of investigation on similar figures. When comparing width and length of a rectangular shape, the book introduced the word "ratio" as a way to relate two quantities. For example, the width was one unit wide and 2 units long, so the ratio "1 to 2" could be used to describe the relationship. It also equivalent "1 to 2" to fraction "1/2." Through a series of computer supported explorations, students were expected to discover the following three principles about similar figures: 1) Their general shapes are the same, 2) Their corresponding angles are the same, and 3) Their corresponding side lengths are related by the same scale factor. The idea of "equal ratio" was explored informally through the last principle: scaling up and down the corresponding side lengths by the same scale factor. Later the term "ratio" was introduced more formally as a way to compare two quantities. The concept of "equal ratio" was mentioned as

"scaling up and down a ratio." The following example was given: "Suppose a shade of purple paint is made using 2 parts red paint to 3 parts blue. You would get the same shade of purple whether you mixed 2 gallons of red paint to 3 gallons of blue paint, 4 gallons of red paint to 6 gallons of blue pain, or 6 gallons of red paint to 9 gallons of blue paint." No rule was given explicitly in the student textbook.

Similar to all three Asian textbook series, *the Mathematics in Contexts* introduced the ideal of "equal ratio" through the context of equivalent fractions. Specially, "ratio table" was introduced as a model use to increase or decrease the serving sizes of recipes, and later become a tool for organizing fraction calculation. Fore example, the idea that the relationship of cups of sugar to cups of flour remained constant could be shown in the ratio table below:

Cups of Flour	1	2	3	4
Cups of Sugar	1/2	1	1 1/2	2

This model was formally introduced to students in the student textbooks with more examples . Several valid operations could be performed on the table, such as halving, doubling, multiplying, adding and subtracting. Notice that, unlike the other five textbook series, the *Mathematics in Context* regarded multiplying both the first term and the second term by a non-zero number as just one of many ways to find equivalent ratios

Application of ratio

In all three Asian textbook series, problems that require direct application of ratio concepts were included right after the units of equivalent ratio. There were remarkable similarities among three Asian textbooks in terms of the type of exercises included in this unit. There were two basic types. The first type gave an explicitly stated ratio relationship between two quantities, and the actual amount of one of those two quantities, the question was to use the ratio relationship to find the actual amount of the second quantity. For example, " The ratio between the number of boys and the number of girls in a school choir is 3:5. There were 21 boys? How many girls?" (from Taiwan series , sixth grade). The second type was to give the ratio relationship between two quantities, and the actual amount of the sum of the two quantities, the question was to use the ratio relationship to find the actual amount of each of the two quantities. For example: " Two brothers shared 1800 Yen. The ratio between the older brother's money and the little brother's money is 3:2. 1) The big brother's money is what fraction of the whole amount of money? 2) How much money will the big brother get? 3) How much money will the little brother have?" (from Japan series, sixth grade). China series also included worked-out example like the above two types. Furthermore, some examples in China textbook series included ratio among three quantities:

such as 2:3:5. However, in U.S. commercial textbook, we only found example of the first type of questions included in the sixth grade textbook, but not the second type.

Different from the three Asian textbook series and the *Math in My World*, the *Connected Mathematics* and the *Mathematics in Context* did not include exercises with simple statement and unique answer that required students to directly apply their ratio concepts. However, the *Connected Mathematics* included much more elaborated exercise to help students link the concept between ratio, fraction and percentage. And great emphasis was placed on selecting the best way to make comparisons based on the purpose of the comparisons. The *Mathematics in Context* also included elaborated contextual problems that helped students to make connections between the expression among ratio, fraction, decimal and percentage. In one exercise that based on the context of speeders vs. total number of drivers, students were asked to find the equivalent fractions, decimals, and percents for 1:2, 1:4, 1:5 and 1:15. The idea that only part-whole ratio can be converted to percentage meaningfully was first introduced in an earlier unit Per Sense was again been emphasized here.

Application of proportion

All the textbook series we examined treated proportion as an equivalence between two ratios. The idea of co-variation: the change of one quantity influenced the change of another quantity was required in solving proportion problems. We have identified the following four major types of problems to organize this part of analysis: 1) stretching and shrinking: these includes scale drawing and map reading, 2) direct proportion including the problems involving familiar rate measurements, 3) inverse proportion, 4) comparative ratio and rate problems.

Stretching and shrinking problems. All textbook series we examined except China series[2] included problems with stretching and shrinking in the units we examined. Among those five textbook series, the *Math in My World* has the shortest time coverage of the topic (1 lesson in grade 5 and 2 lessons in grade 6), while the *Connected Mathematics* devoted a whole unit "Stretching and Shrinking" (22-23 lessons) on this topic. In terms of content coverage, both Taiwan series and the *Math in Context* introduced the idea of stretching and shrinking through changing the size of the unit squares used in drawings. They then guided students to the principle that the resulting figures would have similar shape but larger or smaller size and there is a constant ratio (scale factor) existing between the corresponding sides. Both the *Math in My World* and the *Connected Mathematics* introduced an additional principle about similar figures "the corresponding angles stayed the same" to their students. However, the introduction of this principle in the *Math in My World* was brief while the students were guided through a series of computer-based activities to reach the same conclusion in *Connected Mathematics*.

All textbook series we examined included discussion and exercises on understanding scale such as the one on the map, floor plan or microscope. Most of the

textbook series included simple application such as "Assume the scale of the map is 1:1000000. What is the actual distance between points A and B if the distance between points A and B on the map is 2 cm?" with various degrees of the contextual information. The *Mathematics in Context* had the most elaborated exercises on this topic. For example, one exercise asked students to compared the actual sizes of two drawings obtained from two microscopes that had different magnifying power.

Direct proportion problems. All three Asian textbook series gave this topic significant attention at the sixth grade level. Various examples were given to help students understand the concept of direct proportion "When one quantity grows k times, the other quantity grows k times also," the notation "y=kx," and the graph. Exercises were included to help students differentiate between the situation involving direct proportion (e.g., length vs. perimeter) and the situation not involving direct proportion (e.g., length vs. area).

All six textbook series we examined included some discussion and exercises on rate-related problems, such as speed and density. However, the degree of the complexity of these types of problems varied greatly among these textbook series. The simple ones were missing value type of exercises, such as, "A car drove 140 km in 2 hours. If the speed stayed the same, and it took the car 5 hours to travel from point A to point B. What was the distance between point A and point B?" Several different strategies were shown for solving this type of problems. For example, one solution was to find the speed by dividing the distance, 140 km, by the amount of time, 2 hours, than multiply by 5 hours to get 350 km. The second method was to recognize the existing equal ratio, and find the scale factor between 5 hours and 2 hours, which was 2.5. Then multiply 2.5 by 140 km to get 350 km. The third method was the "cross product." An equation x/5= 140/2 was set up. When cross-multiplying, 2x=140x5=700. Therefore, x= 350. Only the China series and the *Math in my World* showed this cross-multiplying method. The *Mathematics in Context* with its unique "ratio table" introduced a unique way of solving this type of problem. A student could set up a ratio table for the ratio 2 to 140. Doubled it to get 4 to 280. Halved it to get 1 to 70. Then added the ratio of 4 to 280 to the ratio of 1 to 70 to get a new ratio of 5 to 350.

Inverse proportion problems. All three Asian textbook series gave this topic significant attention at the sixth grade level. Various examples were given to help students understand the concept of inverse proportion "When one quantity grows k times, the other quantity grows 1/k times also," the notation "y=k ÷x" and the graph. Exercise were included to help students differentiate between the situation involving inverse proportion (e.g., time vs. speed when the distance was the constant) and the situation not involving an inverse direct proportion (e.g., length vs. area). None of the three U. S. textbook series we examined included this type of problems.

Comparative ratio/rate problems. None of the Asian textbook series we examined included exercises on comparative ratio/rate at the sixth grade level where the main part of the instruction of ratio and proportion were introduced. Japan textbook

had some discussion of this type of question in a fifth grade unit titled "per-unit-quantity," which was before the main introduction of ratio and proportion units in sixth grade. Taiwan textbook series discussed this type of questions at seventh grade level within the formal discussion of algebraic ratio properties. China textbook series did not contain any this type of problems in the elementary or middle school. However, this type of problems was included in all of three U.S. textbook series did. The amount of emphasis varied from one lesson (*Math in My World*), to 15-22 *lessons (Connected Mathematics* and *Mathematics in Context*).

Conclusion

This study attempted to compare the introduction of ratio and proportion concepts in the textbook series from three Asian regions and United States. Similar to the previous studies on international textbook comparison, we also found great differences between the Asian and U.S. textbook series. However, we also noted that the one U.S. commercial textbook series *Math in My World* included this study was more similar to the Asian textbook series in their form of presentations and the structures of the topics than the U.S. reform textbook series. For example, all three Asian textbook series and *Math in My World* used contextual problems to support and motivate the introduction of the concepts and procedures. However those contextual problems tended to be short and specific to the concepts and or techniques. Both the *Connected Mathematics* and the *Mathematics in Context* used elaborated contextual problems with rich contexts and many connected sub-problems to support students to develop their own ideas and strategies among many different concepts. Finally, reasoning and communication were emphasized throughout both textbook series explicitly by require students to write extensively. Certainly, there are differences among three Asian textbook series, between Asian and U.S. commercial textbook series, between U.S. commercial and reform textbook series, and between the two U. S. reform textbook series. Further studies will be needed to examine these differences in depth.

We also noted many differences among these six textbook series in terms of their definitions of ratio, the way equal ratios were introduced and the type of application problems included in the books. For example, the idea of multiplicative comparison was emphasized explicitly by the Asian textbook series but not by the others. Also, *Mathematics in Context* discussed many different ways equal ratios could be generated such as halving, doubling, adding, subtracting, and scale multiplying/dividing, while all the other textbook series discussed only scale multiplying/dividing to generate equal ratios.

There was also significant difference in terms of sequencing. For example, *Connected Mathematics* introduced the idea of similar figures at seventh grade and before the main discussion of ratio and proportion. Yet most of the other textbook series introduced the idea of similar figure after the introduction of ratio and proportion. A future study that focuses on the sequence of certain big mathematical ideas should

have great instructional implication. Finally, with all the emphasis on reasoning and connection in both U.S. reform textbook series, will the students that have gone through either textbook series showed greater ability in these two areas? Future studies are needed to see if we can identify the influence of these textbook differences on student performance.

Endnotes

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2. China textbook series included topics on similar figures at the second part of seventh grade. This topic was discussed at a much more formal level both geometrically and algebraically. Therefore, we did not include it in our discussion.

References

Cai, J. (2001). Improving mathematics learning: Lessons from cross-national studies of U.S. and Chinese students. *Phi Delta Kappan, 82*, 400-405.

Cai, J., Lo. J., & Watanabe, T. (2001). *Intended curricular treatments for students' conceptual understanding in U.S. and Asian school mathematics: A case of arithmetic average*. Paper presented at the Annual Meeting of American Educational Research Association, Seattle, WA; April 10-14, 2001.

Clements, D. H., Jones, K. W., Moseley, L.G., & Schulman, L. (1999). *Math in my world. Teacher's Edition (Grade 5 and Grade 6)*. New York: McGraw-Hill.

Division of Mathematics. (1996). *National unified mathematics textbooks in elementary school.* Beijing: People's Education Press.

Fey, J. T., Fitzgerald, W. M., Friel, S. N., Lappan, G., & Phillips, E. D. (1998). *Connected mathematics*. Menlo Park, CA: Dale Seymour Publications.

Harel, G., Behr, M., Post, T., & Lesh, R. (1991). Variables affecting proportionality. In F. Furinghetti (Ed.), *Proceedings of the Fifteenth Annual Meeting of the International Group for the Psychology of Mathematics Education*, (Vol. 2, pp. 125-132). Assisi, Italy: International Group for the Psychology of Mathematics Education.

Kaput, J., & West, M. (1994). Missing value proportional reasoning problems: Factors affecting informal reasoning patterns. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 61-85). Albany, NY: State University of New York Press.

Lamon, S. (1993). Ratio and proportion: Connecting content and children's thinking. *Journal for Research in Mathematics Education, 24*, 41-61.

Li, Y. P. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education, 31*, 234-241

Lo, J. & Watanabe, T. (1997). Developing ratio and proportion schemes: A story of a fifth grader. *Journal for Research in Mathematics Education, 28*, 216-236.

Mayers, S., & Tajika, H. (1995). A comparison of how textbooks teach mathematical problem solving in Japan and the United States. *American Educational Research Journal, 32*, 443-460.

National Center for Research in Mathematical Sciences Education at the University of Wisconsin/Madison and Freudenthal Institute at the University of Utrecht. (1997-1998). *Mathematics in context*. Britannica.

National Printing Office. (1999). *Elementary school mathematics* (6th ed.). Taipei, Taiwan.

Sammy K., & Liu, J. (1997). A comparative study of selected United States and Japanese first-grade mathematics textbooks. *Focus on Learning Problems in Mathematics, 19*, 1-13.

Tokyo Shoseki. (1998). *Shinhen atarashii sansuu (New elementary school mathematics)*. Tokyo: Author.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 141-161). Reston, VA: National Council of Teachers of Mathematics.

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